Lesson 24.1 and 24.2: Perpendicular and Angle Bisectors

When a point is the same distance from two or more objects, it is equidistant from the objects. The distance to the objects. Triangle congruence theorems can be used to prove theorems about equidistant points.

A **locus** is a set of points that satisfies a given condition. The perpendicular bisector of a segment can be defined as the locus of points in a plane that are equidistant from the endpoints of the segment.

Theorems Distance and Perpendicular Bisectors

THEOREM	HYPOTHESIS	CONCLUSION
6-1-1 Perpendicular Bisector Theorem If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.	$\overleftrightarrow{PM}$ is the ⊥ bisector of $\overline{AB}$	$PA = PB$
6-1-2 Converse of the Perpendicular Bisector Theorem If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.	$DA = DB$	$\overleftrightarrow{DM}$ is the ⊥ bisector of $\overline{AB}$

[Ex 1] Applying the Perpendicular Bisector Theorem and Its Converse

Leave space for work

Find each measure.

- EF
- BC
- RS

YOUR TURN

Find each measure.

1a. Given that line ℓ is the perpendicular bisector of $\overline{DE}$ and $EG = 14.4$, find DG.

1b. Given that $DE = 20.6$, $DG = 30.4$, and $EG = 36.4$, find EF.

Theorems Distance and Angle Bisectors

THEOREM	HYPOTHESIS	CONCLUSION
6-1-3 Angle Bisector Theorem If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.	$\overrightarrow{PB}$ bisects $\angle APC$	$PC = PC$
6-1-4 Converse of the Angle Bisector Theorem If a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle.	$PC = PC$	$\angle APC \cong \angle BPC$

[Ex 2] Applying the Angle Bisector Theorem

Leave space for work

Find each measure.

- LM
- $m\angle ABD$, given that $m\angle ABC = 112°$
- $m\angle TSV$

YOUR TURN

Find each measure.

2a. Given that $\overline{YW}$ bisects $\angle XYZ$ and $WZ = 3.85$, find WX.

2b. Given that $m\angle WYZ = 65°$, $XW = 5.7$, and $ZW = 5.7$, find $m\angle XYZ$.

[Ex 3] Parachute Application

Each pair of suspension lines on a parachute are the same length and are equally spaced from the center of the chute. How do these lines keep the skydiver centered under the parachute?

[Ex 4] Writing Equations of Bisectors in the Coordinate Plane

Write an equation in point-slope form for the perpendicular bisector of the segment with endpoints $P(5, 2)$ and $Q(1, -4)$.

YOUR TURN

Write an equation in point-slope form for the perpendicular bisector of the segment with endpoints $P(5, 2)$ and $Q(1, -4)$.

When three or more lines intersect at one point, the lines are said to be **concurrent**. The point of intersection is called the **point of concurrency**. In the construction, you saw that the three perpendicular bisectors of a triangle are concurrent. This point of concurrency is the **circumcenter of the triangle**.

Theorem 6-2-1 Circumcenter Theorem

The circumcenter of a triangle is equidistant from the vertices of the triangle.

$PA = PB = PC$

The circumcenter can be inside the triangle, outside the triangle, or on the triangle.

Acute triangle | Obtuse triangle | Right triangle

The circumcenter of $\triangle ABC$ is the center of its circumscribed circle. A circle that contains all the vertices of a polygon is **circumscribed** about the polygon.

[Ex 5] Using Properties of Perpendicular Bisectors

$\overline{KP}$, $\overline{PL}$, and $\overline{KL}$ are the perpendicular bisectors of $\triangle GHI$. Find HL.

YOUR TURN Use the diagram above. Find each length.

1a. GH **1b.** GL **1c.** LI

[Ex 6] Finding the Circumcenter of a Triangle

Find the circumcenter of $\triangle RST$ with vertices $R(-6, 4)$, $S(6, 4)$, and $T(0, 0)$.

Step 1 Graph the triangle.

Step 2 Find two lines that are perpendicular bisectors of the triangle.
Since two sides of the triangle lie along the axes, use the graph to find the perpendicular bisectors of these two sides. The perpendicular bisector of $\overline{RS}$ is $x = -3$, and the perpendicular bisector of $\overline{ST}$ is $y = 2$.

Step 3 Find the intersection of the two equations.
The lines $x = -3$ and $y = 2$ intersect at $(-3, 2)$, the circumcenter of $\triangle RST$.

YOUR TURN Find the circumcenter of $\triangle GHM$ with vertices $G(6, -5)$, $H(4, 4)$, and $M(8, 0)$.

A triangle has three angles, so it has three angle bisectors. The angle bisectors of a triangle are also concurrent. This point of concurrency is the **incenter of the triangle**.

Theorem 6-2-2 Incenter Theorem

The incenter of a triangle is equidistant from the sides of the triangle.

$PX = PY = PZ$

Unlike the circumcenter, the incenter is always inside the triangle.

Acute triangle | Obtuse triangle | Right triangle

The incenter is the center of the triangle's inscribed circle. A circle is **inscribed** in a polygon if it intersects each line that contains a side of the polygon at exactly one point.

[Ex 7] Using Properties of Angle Bisectors

$\overline{PT}$ and $\overline{RT}$ are angle bisectors of $\triangle JKL$. Find each measure.

- the distance from T to $\overline{KL}$
- $m\angle TRL$

YOUR TURN

$\overline{QS}$ and $\overline{RS}$ are angle bisectors of $\triangle PQR$. Find each measure.

2a. the distance from S to $\overline{PQ}$

2b. $m\angle PQS$