

Lesson 29.1: Area of Regular Polygons

[EX. 1: CIRCLES]

Circumference and Area Circle

A circle with diameter d and radius r has circumference $C = \pi d$ or $C = 2\pi r$ and area $A = \pi r^2$.



Finding Measurements of Circles

Find each measurement.

- A** the area of $\odot P$ in terms of π
- B** the radius of $\odot X$ in which $C = 24\pi$ in.
- C** the circumference of $\odot S$ in which $A = 9x^2\pi$ cm²

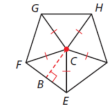


YOUR TURN

- 1. Find the area of $\odot A$ in terms of π in which $C = (4x - 6)\pi$ m.

[EX. 2: AREAS OF REGULAR POLYGONS]

The **center of a regular polygon** is equidistant from the vertices. The **apothem** is the distance from the center to a side. A **central angle of a regular polygon** has its vertex at the center, and its sides pass through consecutive vertices. Each central angle measure of a regular n -gon is $\frac{360^\circ}{n}$.



Regular pentagon $DEFGH$ has center C , apothem BC , and central angle $\angle DCE$.

To find the area of a regular n -gon with side length s and apothem a , divide it into n congruent isosceles triangles.

area of each triangle: $\frac{1}{2}as$
 total area of the polygon: $A = n\left(\frac{1}{2}as\right)$, or $A = \frac{1}{2}aP$ The perimeter is $P = ns$.

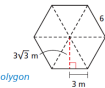
Area Regular Polygon

The area of a regular polygon with apothem a and perimeter P is $A = \frac{1}{2}aP$.



Find the area of each regular polygon. Round to the nearest tenth.

- A** a regular hexagon with side length 6 m
 The perimeter is $6(6) = 36$ m. The hexagon can be divided into 6 equilateral triangles with side length 6 m. By the 30° - 60° - 90° Triangle Theorem, the apothem is $3\sqrt{3}$ m.
 $A = \frac{1}{2}aP$ Area of a regular polygon



- B** a regular pentagon with side length 8 in.
Step 1 Draw the pentagon. Draw an isosceles triangle with its vertex at the center of the pentagon. The central angle is $\frac{360^\circ}{5} = 72^\circ$. Draw a segment that bisects the central angle and the side of the polygon to form a right triangle.



Step 2 Use the tangent ratio to find the apothem.
 $\tan 36^\circ = \frac{4}{a}$ The tangent of an angle is $\frac{\text{opp. leg}}{\text{adj. leg}}$.
 $a = \frac{4}{\tan 36^\circ}$ Solve for a .

Step 3 Use the apothem and the given side length to find the area.
 $A = \frac{1}{2}aP$ Area of a regular polygon
 $A = \frac{1}{2}\left(\frac{4}{\tan 36^\circ}\right)(40)$ The perimeter is $8(5) = 40$ in.
 $A \approx 110.1$ in² Simplify. Round to the nearest tenth.

YOUR TURN

- 3. Find the area of a regular octagon with a side length of 4 cm.