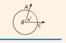
Lesson 30.2: Arcs and Chords

A **central angle** is an angle whose vertex is the center of a circle. An **arc** is an unbroken part of a circle consisting of two points called the endpoints and all the points on the circle between them.

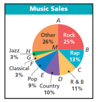
Arcs and Their Measure

ARC	MEASURE	DIAGRAM
A **minor arc** is an arc whose points are on or in the interior of a central angle.	The measure of a minor arc is equal to the measure of its central angle. $m\overset{\frown}{AC} = m\angle ABC = x°$	
A **major arc** is an arc whose points are on or in the exterior of a central angle.	The measure of a major arc is equal to 360° minus the measure of its central angle. $m\overset{\frown}{ADC} = 360° - m\angle ABC = 360° - x°$	
If the endpoints of an arc lie on a diameter, the arc is a **semicircle**.	The measure of a semicircle is equal to 180°. $m\overset{\frown}{EFG} = 180°$	

Minor arcs may be named by two points. Major arcs and semicircles must be named by three points.

[Ex. 1] ***Data Application***

The circle graph shows the types of music sold during one week at a music store. Find $m\overset{\frown}{BC}$.



YOUR TURN **Use the graph to find each of the following.**

1a. $m\angle FMC$ **1b.** $m\overset{\frown}{AHB}$ **1c.** $m\angle EMD$

Adjacent arcs are arcs of the same circle that intersect at exactly one point. $\overset{\frown}{RS}$ and $\overset{\frown}{ST}$ are adjacent arcs.

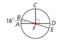
Postulate 12-2-1 Arc Addition Postulate

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

$m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$

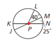
[Ex. 2] **Using the Arc Addition Postulate**

Find $m\overset{\frown}{CDE}$



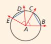
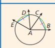
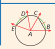
YOUR TURN **Find each measure.**

2a. $m\overset{\frown}{JKL}$ **2b.** $m\overset{\frown}{LJN}$



Within a circle or congruent circles, **congruent arcs** are two arcs that have the same measure. In the figure, $\overset{\frown}{ST} \cong \overset{\frown}{UV}$.

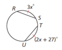
Theorem 12-2-2

THEOREM	HYPOTHESIS		CONCLUSION
In a circle or congruent circles: (1) Congruent central angles have congruent chords.		$\angle EAD \cong \angle BAC$	$\overline{DE} \cong \overline{BC}$
(2) Congruent chords have congruent arcs.		$\overline{ED} \cong \overline{BC}$	$\overset{\frown}{DE} \cong \overset{\frown}{BC}$
(3) Congruent arcs have congruent central angles.		$\overset{\frown}{ED} \cong \overset{\frown}{BC}$	$\angle DAE \cong \angle BAC$

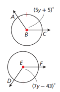
[Ex.3] **Applying Congruent Angles, Arcs, and Chords**

Find each measure.

A $\overline{RS} \cong \overline{TU}$. Find $m\overset{\frown}{RS}$.

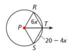


B $\odot B \cong \odot E$, and $\overset{\frown}{AC} \cong \overset{\frown}{DF}$. Find $m\angle DEF$.

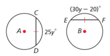


YOUR TURN **Find each measure.**

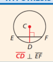
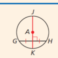
3a. $\overrightarrow{PT}$ bisects $\angle RPS$. Find RT.



3b. $\odot A \cong \odot B$, and $\overline{CD} \cong \overline{EF}$. Find $m\overset{\frown}{CD}$.

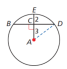


Theorems

THEOREM	HYPOTHESIS	CONCLUSION
12-2-3 In a circle, if a radius (or diameter) is perpendicular to a chord, then it bisects the chord and its arc.	 $\overline{CD} \perp \overline{EF}$	$\overline{CD}$ bisects $\overline{EF}$ and $\overset{\frown}{EF}$.
12-2-4 In a circle, the perpendicular bisector of a chord is a radius (or diameter).	 $\overline{JK}$ is $\perp$ bisector of $\overline{GH}$.	$\overline{JK}$ is a diameter of $\odot A$.

[Ex. 4] **Using Radii and Chords**

Find BD.



YOUR TURN **4.** Find QR to the nearest tenth.