7.5 Linear Programming

Georgia Performance Standard(s)
MM3A6b

Goal • Solve linear programming problems.

Your Notes

VOCABULARY

Linear programming a process of maximing a linear objective function

Objective function Gives a quantity that is to maximized (or minimized) and is subject to constraints.

Constraints The linear inequalities that form a system in a linear programming problem

Feasible region The intersection of all the graphs of the constraints in a linear programming problem

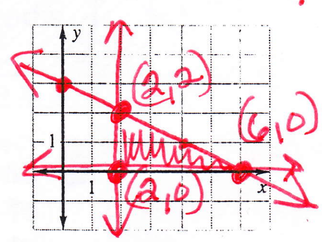
Example 1 ***Solve a linear programming problem***

Find the minimum value and the maximum value of the objective function $C = 2x + 5y$ subject to the following constraints.

$x \geq 2, y \geq 0, 4x + 8y \leq 24$

$4x + 8y \leq 24$
$8y \leq -4x + 24$
$y = -\frac{1}{2}x + 3$

Solution



1. **Graph** the system of constraints. The coordinates of the vertices of the feasible region are (2,0), (2,2), and (6,0).

2. **Evaluate** the function $C = 2x + 5y$ at each vertex.

At (2, 0): $C = 2(2) + 5(0) = 4$ ← minimum
At (2, 2): $C = 2(2) + 5(2) = 14$ ← maximum
At (6, 0): $C = 2(6) + 5(0) = 12$

The minimum value of C is 4. It occurs when $x =$ 2 and $y =$ 0. The maximum value of C is 14. It occurs when $x =$ 2 and $y =$ 2.

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Your Notes

Example 2 ***Solve a linear programming problem***

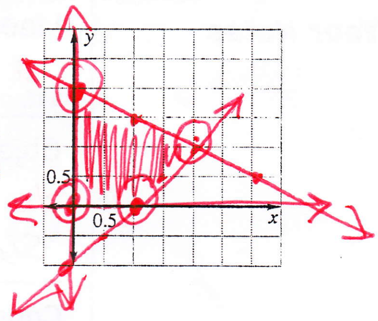
Find the minimum value and the maximum value of the objective function $C = 3x + 4y$ subject to the following constraints.

$-y \le -x + 1$

$x \ge 0, y \ge 0, x + 2y \le 4, x - y \le 1$ $y \ge x - 1$

$y \le -\frac{1}{2}x + 2$

Solution



1. Graph the system of constraints. Find the coordinates of the vertices of the feasible region by solving the systems of two linear equations. The solution of the system

$x + 2y = 4$

$x - y = 1$

gives the vertex (2, 1). The other three vertices are (0, 0), (1, 0), and (0, 2).

2. Evaluate the function $C = 3x + 4y$ at each vertex.

At (0, 0): $C = 3(0) + 4(0) = 0$ ← minimum

At (1, 0): $C = 3(1) + 4(0) = 3$

At (2, 1): $C = 3(2) + 4(1) = 10$ ← maximum

At (0, 2): $C = 3(0) + 4(2) = 8$

The minimum value of C is 0. It occurs when $x =$ 0 and $y =$ 0. The maximum value of C is 10. It occurs when $x =$ 2 and $y =$ 1.

✔ ***Checkpoint*** **Find the minimum and maximum values of the objective function $C = 6x + 3y$ subject to the given constraints.**

$(0,5) \to 6(0)+3(5)=15$

$(4,3) \to 6(4)+3(3)=33$

$(0,0) \to 6(0)+3(0)=0$

$(6,0) \to 6(6)+3(0)=36$

Homework

1. $x \ge 0, y \ge 0,$

$x + 2y \le 10, 3x + 2y \le 18$

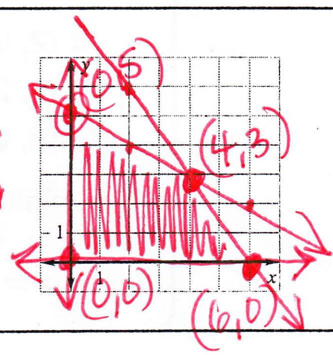
$-x \quad - \quad -x$

$2y \le -x + 10$

$y \le -\frac{1}{2}x + 5$

$2y \le -3x + 18$

$y \le -\frac{3}{2}x + 9$



min @ (0,0) max @ (6,0)

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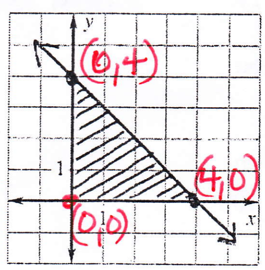
Name ______________________________ Date ____________________

LESSON 7.5

Practice

Find the minimum and maximum values of the objective function for the given feasible region.

1. $C = 3x + y$

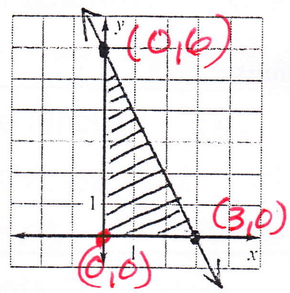


$(0,0) \to 3(0)+0 = 0$ min

$(4,0) \to 3(4)+0 = 12$ max

$(0,4) \to 3(0)+4 = 4$

2. $C = x + 2y$

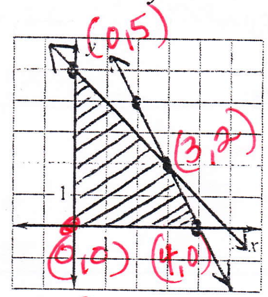


$(0,6) \to 0+2(6) = 12$ max

$(0,0) \to 0+2(0) = 0$ min

$(3,0) \to 3+2(0) = 3$

3. $C = 2x + 3y$



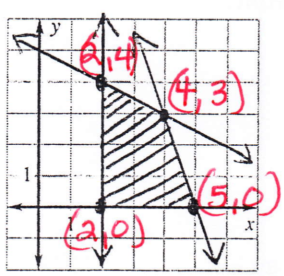
$(0,5) \to 2(0)+3(5) = 15$ max

$(0,0) \to 2(0)+3(0) = 0$ min

$(4,0) \to 2(4)+3(0) = 8$

$(3,2) \to 2(3)+3(2) = 12$

4. $C = 4x + 2y$



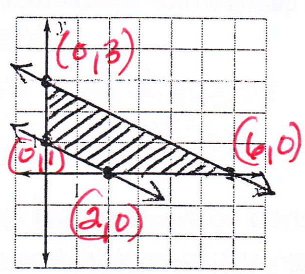
$(2,0) \to 4(2)+2(0) = 8$ min

$(2,4) \to 4(2)+2(4) = 16$

$(5,0) \to 4(5)+2(0) = 20$

$(4,3) \to 4(4)+2(3) = 22$ max

5. $C = 5x - 3y$



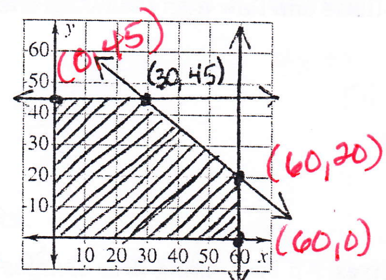
$(0,3) \to 5(0)-3(3) = -9$ min

$(0,1) \to 5(0)-3(1) = -3$

$(2,0) \to 5(2)-3(0) = 10$

$(6,0) \to 5(6)-3(0) = 30$ max

6. $C = 10x + 7y$



$(0,45) \to 10(0)+7(45) = 315$

$(30,45) \to 10(30)+7(45) = 615$

$(60,20) \to 10(60)+7(20) = 740$ max

$(60,0) \to 10(60)+7(0) = 600$

$(0,0) \to 10(0)+7(0) = 0$ min

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