

## REVIEW

COMPLETE and CHECK ANSWERS ONLINE

**Formulas.** Write the formula for each term below and explain each part of the formula.

<p><b>Explicit formula for a geometric sequence</b></p> $a_n = a_1 \cdot r^{n-1}$ <p>↑      ↑      ↑      ↑  final value   1st term in sequence   common ratio   term you want</p>	<p><b>Exponential growth</b></p> $Y = a(1+r)^t$ <p>↑      ↑      ↑      ↑  final value   orig value   rate written as decimal   time</p>	<p><b>Exponential decay</b></p> $y = a(1-r)^t$ <p>↑      ↑      ↑      ↑  final value   orig value   rate written as decimal   time</p>
<p><b>Half-life</b></p> $A = P(.5)^{t/h}$ <p>↑      ↑      ↑  final value   orig value   time ÷ half life</p>	<p><b>Compound interest</b></p> $A = P\left(1 + \frac{r}{n}\right)^{nt}$ <p>↑      ↑      ↑      ↑      ↑  final value   orig value   rate written as decimal   # of times compounded per year   time</p>	

Identify each of the following functions as exponential growth or decay. Then give the rate of growth or decay as a percent.

1.  $y = 710(1.289)^t$  growth       $1.289 - 1 = .289 = 28.9\%$

2.  $y = 31(0.62)^t$  decay       $1 - .62 = .38 = 38\%$

3.  $y = a\left(\frac{3}{5}\right)^t$  decay       $\frac{3}{5} = .6$        $1 - .6 = .4 = 40\%$

4.  $y = a\left(\frac{10}{4}\right)^t$  growth       $\frac{10}{4} = 2.5$        $2.5 - 1 = 1.5 = 150\%$

Write an exponential growth or decay function to model each situation. Then find the value of the function after the given amount of time.

5. Find a bank account balance if the account starts with \$100, has an annual rate of 4%, and the money left in the account for 12 years.

$$100(1 + .04)^{12} \approx 160.10$$

6. In 1985, there were 285 cell phone subscribers in the small town of Centerville. The number of subscribers increased by 75% per year after 1985. How many cell phone subscribers were in Centerville in 1994?

$$285(1 + .75)^9 \approx 43872$$

7. Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus doubling. If we start with only one bacteria which can double every hour, how many bacteria will we have by the end of one day?

1, 2, 4, 8, ...  $a_n = 1 \cdot 2^{n-1}$   
 $a_{24} = 1 \cdot 2^{23} = 8388608$

8. Each year the local country club sponsors a tennis tournament. Play starts with 128 participants. During each round, half of the players are eliminated. How many players remain after 5 rounds?

128, 62, 31, ...  $a_n = 128 \cdot (.5)^{n-1}$   
 $a_5 = 128 \cdot (.5)^4 = 8$

9. The population of Winnemucca, Nevada, can be modeled by  $P = 6191(1.04)^t$  where  $t$  is the number of years since 1990. What was the population in 1990? By what percent did the population increase by each year?

1990 - 6,191  
% inc  $\rightarrow$  4%

10. You have inherited land that was purchased for \$30,000 in 1960. The value of the land increased by approximately 5% per year. What is the approximate value of the land in the year 2011?

$$30000(1 + .05)^{51} = 361223.09$$

11. During normal breathing, about 12% of the air in the lungs is replaced after one breath. Write an exponential decay model for the amount of the original air left in the lungs if the initial amount of air in the lungs is 500 mL. How much of the original air is present after 240 breaths?

$$500(1 - .12)^{240} \\ = .0000000000237$$

12. An adult takes 400 mg of ibuprofen. Each hour, the amount of ibuprofen in the person's system decreases by about 29%. How much ibuprofen is left after 6 hours?

$$400(1 - .29)^6 = 51.24$$

13. You deposit \$1600 in a bank account. Find the balance after 3 years for each of the following situations:

a. The account pays 2.5% annual interest compounded monthly.  $1600\left(1 + \frac{.025}{12}\right)^{36}$   
1724.48

b. The account pays 1.75% annual interest compounded quarterly.  $1600\left(1 + \frac{.0175}{4}\right)^{12}$   
1686.05

c. The account pays 4% annual interest compounded yearly.  $1600\left(1 + \frac{.04}{1}\right)^3$   
1799.78

14. You buy a new computer for \$2100. The computer decreases by 50% annually. When will the computer have a value of \$600?

$$2100(1 - .5)^x = 600$$

about 2 years

15. The foundation of your house has about 1,200 termites. The termites grow at a rate of about 2.4% per day. How long until the number of termites doubles?

$$1200(1 + .024)^x = 2400$$

$$(1.024)^x = 2$$

about 30 years

16. The half-life of Zn-71 is 2.4 minutes. If one had 100.0 g at the beginning, how many grams would be left after 7.2 minutes has elapsed?

$$100(.5)^{7.2 \div 2.4}$$

12.5

17. The half life of Cs-137 is 30.2 years. If the initial mass of the sample is 1.00kg, how much will remain after 151 years?

$$1(.5)^{151 \div 30.2}$$

.03125

18. Carbon-14 has a half life of 5730 years. Consider a sample of fossilized wood that when alive would have contained 24g of C-14. It now contains 1.5g. How old is the sample?

4 half lives  $\times$  5730 = 22920 years in a half life  $\uparrow$  age

$$\frac{24(.5)^{t \div 5730}}{24} = \frac{1.5}{24}$$

$$.5^{t \div 5730} = .0625$$

# of half lives  $\rightarrow$

$$.5^4 = .0625$$

19. Find the terms  $a_2$ ,  $a_3$ ,  $a_4$  and  $a_5$  of a geometric sequence if  $a_1 = 10$  and the common ratio  $r = -1$ .

$$a_n = 10 \cdot (-1)^{n-1}$$

$$\begin{matrix} 10, & -10, & 10, & -10, & 10 \\ a_1 & a_2 & a_3 & a_4 & a_5 \end{matrix}$$

20. Find the 10th term of a geometric sequence if  $a_1 = 45$  and the common ratio  $r = 0.2$ .

$$a_{10} = 45 \cdot 0.2^9 = 23040$$

21. Find  $A_{20}$  of a geometric sequence if the first few terms of the sequence are given by

$$-1/2, 1/4, -1/8, 1/16, \dots$$

$$a_{20} = -1/2 \cdot (-1/2)^{19} = .000000954$$

22. A ball is dropped from a height of 8 feet. The ball bounces to 80% of its previous height with each bounce. How high (to the nearest tenth of a foot) does the ball bounce on the fifth bounce?

$$8(1 - .8)^5 = .00256$$

23. The third term of a geometric sequence is 3 and the sixth term is  $1/9$ . Find the first term.

$$\begin{matrix} \textcircled{27} & 9 & 3 & 1 & 1/3 & 1/9 \\ & \swarrow & \searrow & & & \\ & a_2 & a_3 & a_4 & a_5 & a_6 \\ & & \times 3 & \times 1/3 & & \\ a_1 & a_2 & & & & \end{matrix} \quad a_1 = 27$$

24. In a certain region, the number of highway accidents increased by 20% over a four year period. How many accidents were there in 2006 if there were 5120 in 2002? Hint: When the percent increases, you want the original 100% plus the additional 20%.

- a. Write an explicit formula for the sequence. Explain where you found the numbers you are putting in the formula.

$$5120(1 + .2)^n$$

- b. Identify the value of  $n$  and explain where you found it. Use the explicit formula to solve the problem.

$$n = 4 \\ \downarrow \\ 2006 - 2002$$

$$5120(1 + .2)^4 = 10617$$

25. A house worth \$350,000 when purchased was worth \$335,000 after the first year and \$320,000 after the second year. If the economy does not pick up and this trend continues, what will be the value of the house after 6 years?

$$350000(1 - .04)^x$$

$$350,000(1 - .04)^6 = 273965.23$$

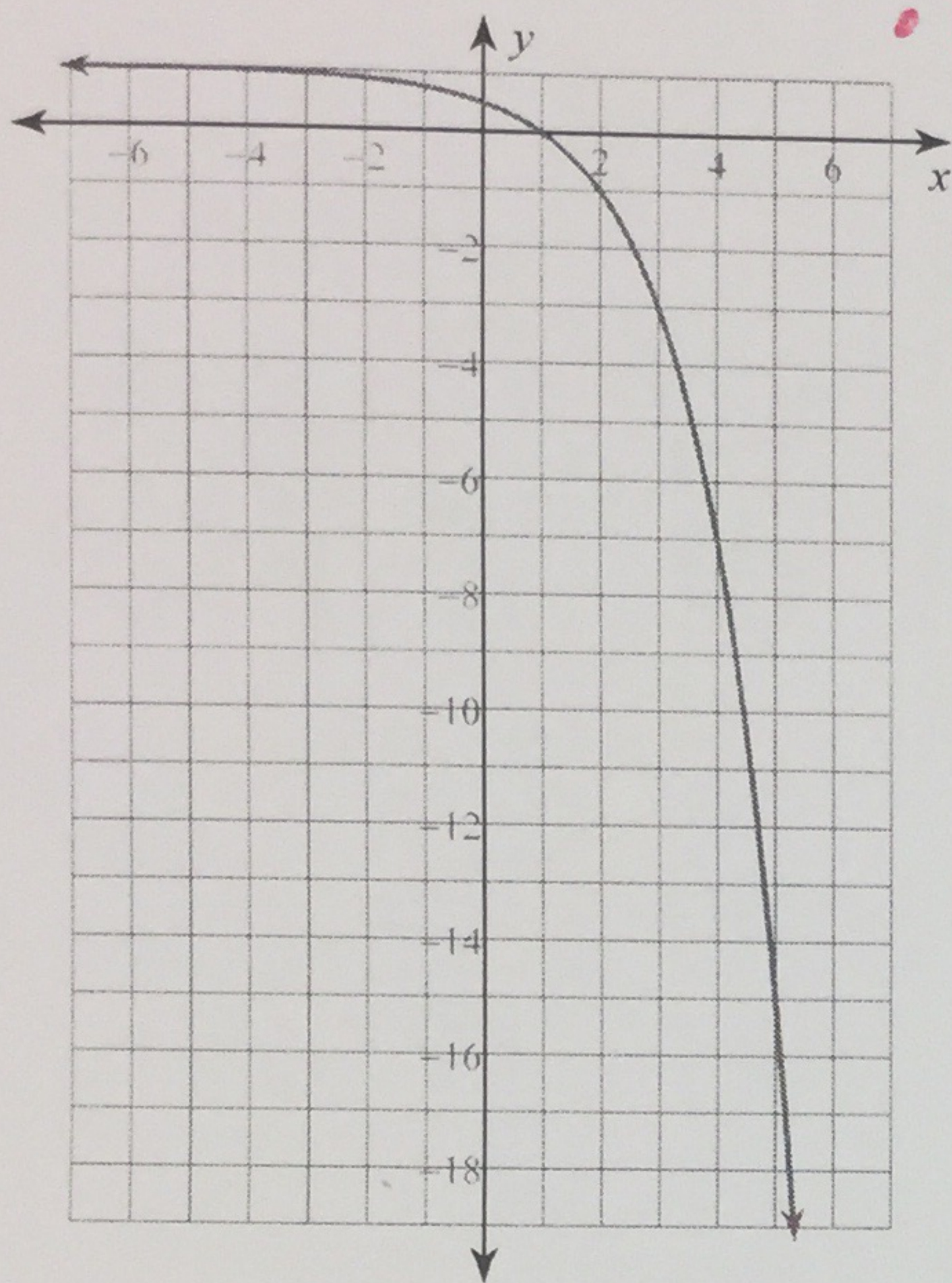
## Graphs

Date \_\_\_\_\_

Period \_\_\_\_\_

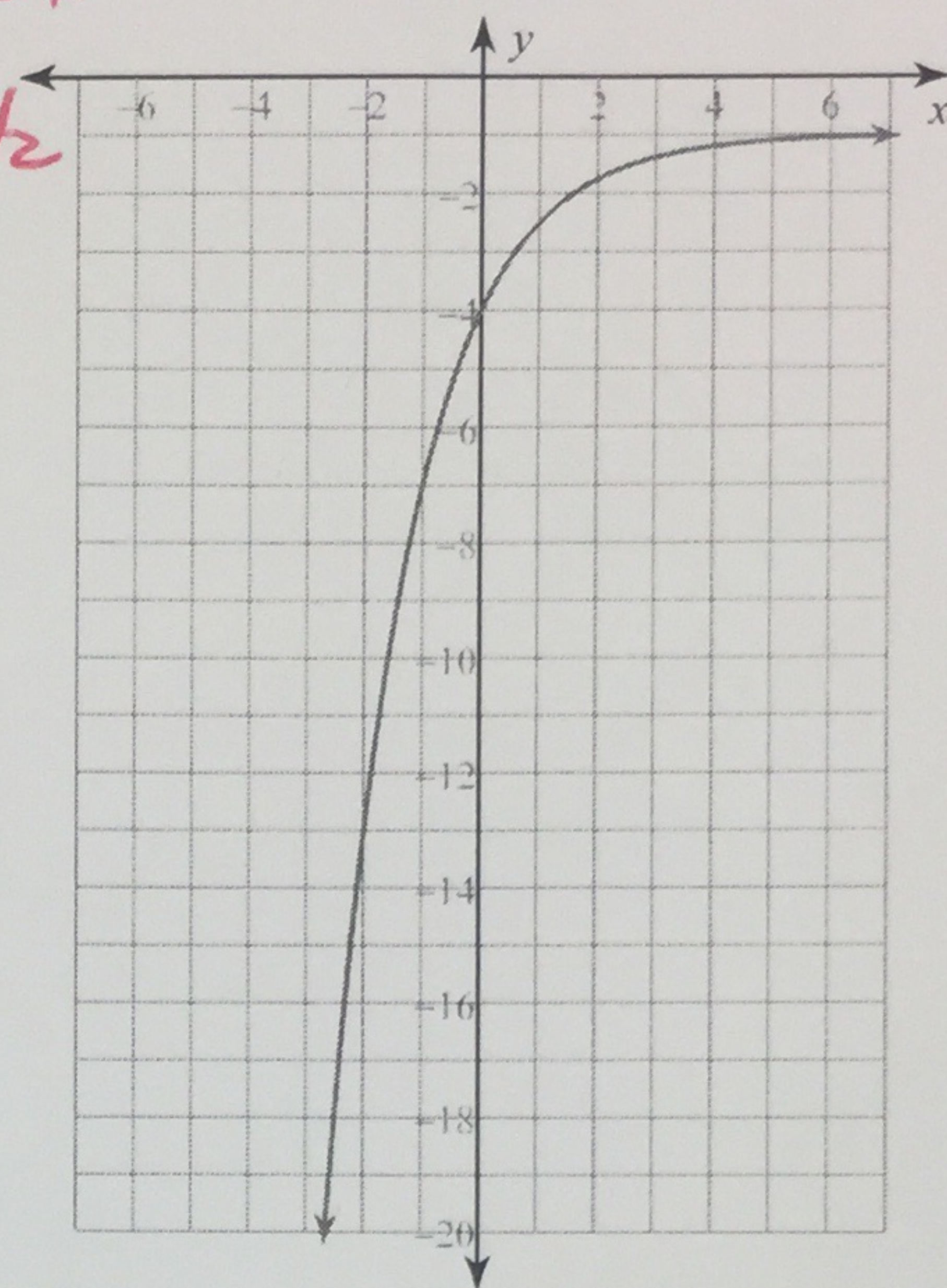
Sketch the graph of each function. Then describe any transformations and whether it is a growth or decay function.

$$1) y = -\frac{1}{2} \cdot 2^x + 1$$



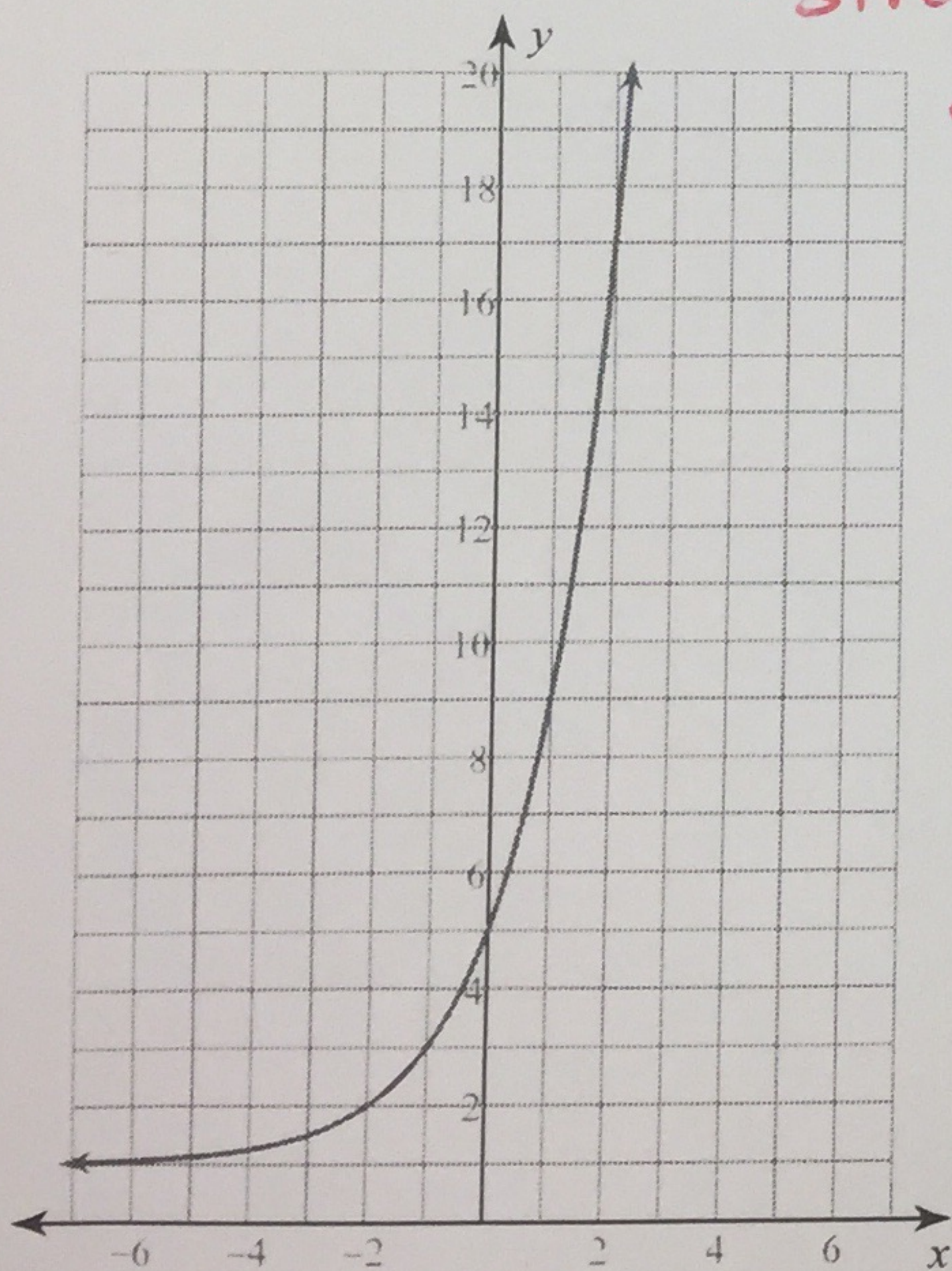
- growth
- reflect across x
- shrink by  $\frac{1}{2}$
- shift up 1

$$2) y = -3 \cdot \left(\frac{1}{2}\right)^x - 1$$



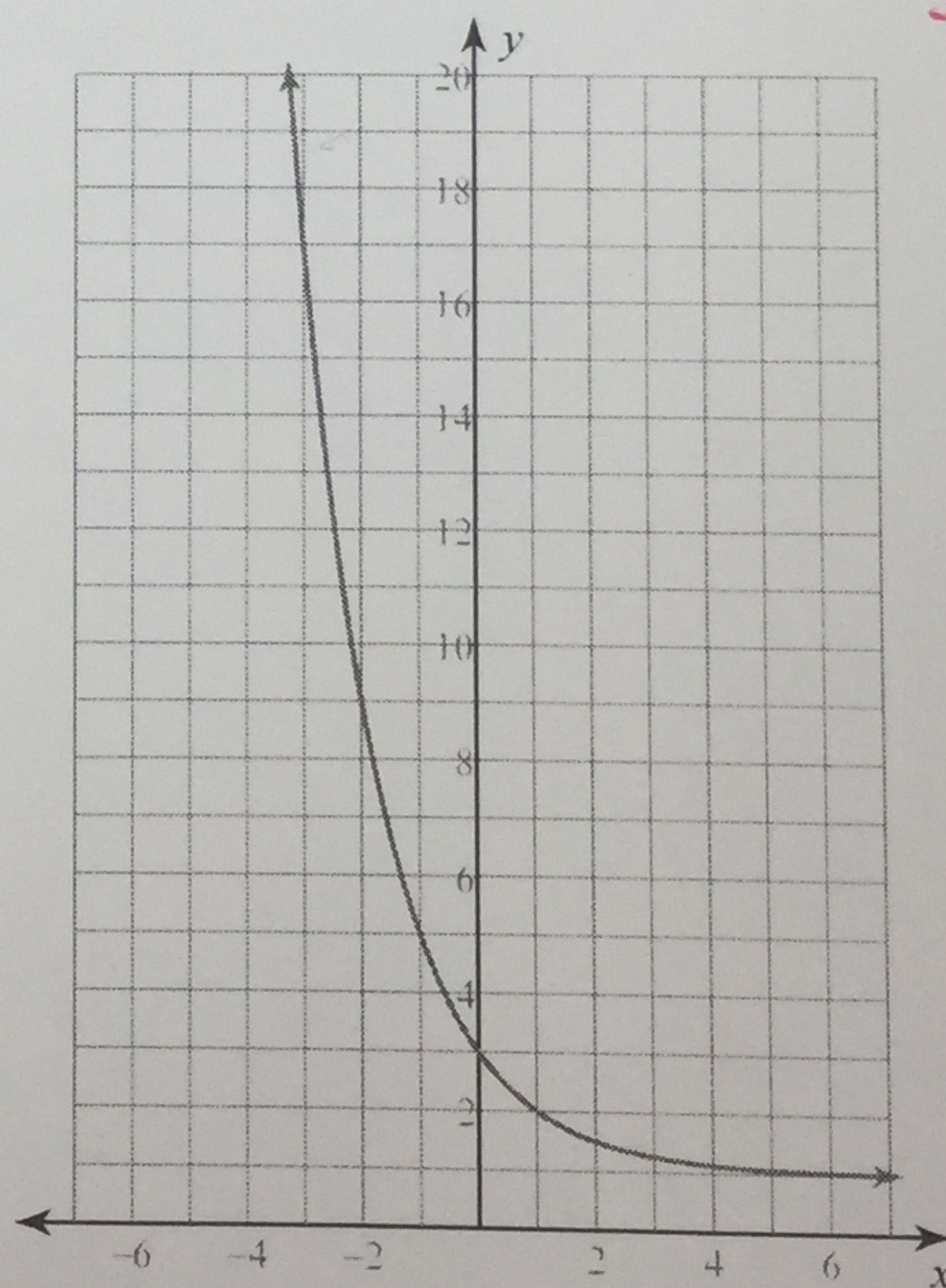
- decay
- reflect across x
- stretch by 3
- shift down 1

$$3) y = 4 \cdot 2^x + 1$$



- growth
- stretch by 4
- shift up 1

$$4) y = 2 \cdot \left(\frac{1}{2}\right)^x + 1$$



- decay
- stretch by 2
- shift up 1