Lesson 6: Rules of Exponents

Vocab
- *Base: big number, factor that is being multiplied
- *Exponent: small number, the number of times the base is multiplied

Quick Review
$5^3 = 5 \times 5 \times 5$
$2 \times 2 \times 2 \times 2 \times 2 = 2^5$

Ex 1: Multiplying (works when bases are the same)
Algebraic: $a^n \times a^m = a^{n+m}$ Numeric: $5^2 \times 5^3 = 5^5$

-Use the multiplicative rule to simplify.
a) $4^2 \times 4^3 = 4^5$
b) $s^4 \times s^2 = s^6$

Ex. 2: Dividing (works when bases are the same)
Algebraic: $\frac{a^n}{a^m} = a^{n-m}$ Numeric: $\frac{7^5}{7^3} = 7^2$

-Use the division rule to simplify.
a) $\frac{15^5}{15^3} = 15^2$ b) $\frac{z^4}{z^3} = z^1$

Ex. 3: Power to a Power (base inside parenthesis)
Algebraic: $(a^n)^m = a^{n \cdot m}$ Numeric: $(7^2)^3 = 7^6$

-Use the power to a power rule to simplify
a) $(8^3)^5$
b) $(14^4)^3$

[SOME MORE RULES FOR EXPONENTS]

1) You cannot have negative exponents
-to get rid of them, flip them to the other side of the fraction, the exponent becomes positive
Ex 1) $2^{-3} = \frac{1}{2^3}$ Ex 2) $\frac{5}{3^{-2}} = \frac{3^2}{2}$

Practice:
a) $4^{-3} =$
b) $\frac{5}{6^{-4}} =$

2) Anything to a power of zero always equals 1
Ex 1) $234^0 = 1$ Ex 2) $(8^2 \times 3^7)^0 = 1$

Practice:
a) $27^0 = 1$
b) $(8a^3)^0 = 1$

3) If the bases aren't the same, work each part out separately then solve
Ex 1) $2^3 \times 3^2 \rightarrow 8 \times 9 = 72$
Ex 2) $2^3 + 3^2 \rightarrow 8 + 9 = 17$

Practice:
a) $5^2 \times 10^2 = 25 \times 100 = 2500$
b) $5^2 + 10^2 = 25 + 100 = 125$

[More challenging problems]

* Remember that the exponent goes with the base it is attached to
* Numbers go with numbers if there are variables also involved

1) $5x^2 \cdot 3x^3 = 15x^5$
2) $x^7 \cdot 2x^4 = 2x^{11}$
3) $(4x^3)^2 = 4^1 \cdot x^6 = \frac{1}{16}$
4) $(2x)^3 = 8x^3$
5) $\frac{4x^{-1}}{2x^2} = \frac{2}{x^3}$
6) $\frac{x^{-2}}{3x^3} = \frac{1}{3x^5}$
7) $\frac{(2^{-3})^2}{2^{-4}} = \frac{2^6}{2^{-4}} = 2^{10} = \frac{1}{2^{10}}$
8) $\frac{(2 \cdot 2^{-3})^2}{(2^2)^3} = \frac{2^{-4}}{2^6} = 2^{-10} = \frac{1}{2^{10}}$
9) $\frac{x^2 \cdot x^4}{x^3} = \frac{x^6}{x^3} = x^3 = \frac{1}{x^{-3}}$
10) $\frac{2^{-4}}{2^2 \cdot 2^{-3}} = \frac{2^{-4}}{2^{-1}} = 2^{-3} = \frac{1}{2^3}$
11) $\frac{y^6}{(y^{-2})^3} = \frac{y^6}{y^{-6}} = y^{12}$
12) $\frac{2^7}{2^6} = 2^1 = 2$

Lesson 7: Scientific Notation
- A way of writing very large or very small numbers

[Writing Standard Form to Scientific Notation Form]
- Pull out the numbers before all the zeros start
- There can only be a 1 digit number in front of the decimal and it must be a number from 1-9
- Count how many spaces are between each decimal, this is your exponent

Ex 1: 3,320,400,000,000 $\rightarrow 2.3204 \times 10^{12}$ (Big Numbers have positive exponents)

Ex 2: 0.0000023204 $\rightarrow 2.3204 \times 10^{-6}$ (Small Numbers have negative exponents)

Practice:
a) 32,480,100,000 $3.248\,01 \times 10^{10}$
b) 0.000000432502 4.3502×10^{-7}

[Writing in Standard Form from Scientific Notation Form]
- working backwards
- positive exponent -> move decimal to the right
- negative exponent -> move decimal to the left

Ex 1: 4.562×10^6 $\rightarrow 4,562,000$
Ex 2: 4.562×10^{-6} $\rightarrow 0.000004562$

Practice:
a) $2.34 \times 10^{-4} = .000234$
b) 7.5438×10^5 754,380.00

[Adding and Subtracting with Scientific Notation]
- must have a 'common exponent'
- exponent is always the larger exponent (most value)
- move the decimal the number of places between the highest value and the lowest value
- make sure your final answer is written in proper scientific notation form

1) $(5.5 \times 10^3) + (4.3 \times 10^3) =$
9.8×10^3
2) $(2.3 \times 10^{-5}) - (6 \times 10^{-6}) =$
17×10^{-6}
3) $(6 \times 10^2) + (3.3 \times 10^3) =$
3.1×10^4
4) $(7.2 \times 10^2) - (5.3 \times 10^1) =$
6.93×10^2
5) $(9.2 \times 10^3) - (8.4 \times 10^3) =$ 8×10^2

[Multiplying and Dividing in Scientific Notation]
- multiply or divide the regular numbers
- if multiplying: add the exponents
- if dividing: subtract the exponents
- make sure your final answer is written in proper scientific notation form

1) $(3.5 \times 10^2) \times (4 \times 10^3) =$
14×10^5
2) $(8 \times 10^6) / (3 \times 10^2) =$
3×10^4
3) $(5 \times 10^{-3}) \times (7 \times 10^{-5}) =$
$35 \times 10^{-8} = 3.5 \times 10^{-7}$
4) $(7.5 \times 10^6) / (2.5 \times 10^4) =$
3×10^2
5) $(4.5 \times 10^3) / (2 \times 10^5) =$ $.00225 \rightarrow 2.25 \times 10^{-3}$